THEORETICAL AND EXPERIMENTAL INVESTIGATION OF CONDENSATION IN A CENTERED RAREFACTION WAVE

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The analysis of the process of spontaneous condensation in one-dimensional formulation is dealt with adequately in many papers. However, in reality supersonic flows are not one-dimensional. The most striking effect of two-dimensionality is manifested in two-phase flows, for example in nozzles, inclined sections of jet turbine grills and rarefaction waves. The investigation of these flows, both in the experimental and theoretical aspect, is a complex problem for which a solution has been found only recently. The results are given in this paper of a theoretical and experimental investigation of spontaneous condensation of water vapor in a centered rarefaction wave formed by flow around a protuberant angle by a hypersonic stream.

1. The theoretical investigation of condensation in a centered rarefaction wave is carried out by the procedure in [1]. The difference consists in that in this case the expansion of pure water vapor is investigated (the mass concentration of the carrier gas is assumed equal to zero). In addition to this, the temperature (condensation) lag of the droplets and gas is taken into account.

We locate the origin of the Cartesian system of coordinates at the point of deflection. We direct the axis x along the velocity on the initial characteristic line emerging from the apex of the angle, and the axis y along the normal to it. We shall refer all parameters of dimensionality of length to a certain characteristic distance from the angular point along the axis y.

For the rate of growth of a droplet, we use Knudsen's formula

$$\frac{dr}{dt} = \frac{\alpha}{(2\pi RT)^{1/2}}\left[p - \left(\frac{T}{T^\circ}\right)^{1/2} p_s(T^\circ)\right] \tag{1.1}$$

where r is the radius of the drop; t is the time; α is the coefficient of condensation; T and T° are the temperatures of the gas and droplet, respectively; R is the gas constant; p is the pressure and $p_s(T)$ is the saturated vapor pressure above the plane surface of the phase transition.

We shall find the temperature of the drop from the equation representing the energy balance of the condensing, reflected, and vaporizing molecules. If we denote the coefficient of thermal adaptation by β, the isentropy index by $\varkappa$, and the heat of condensation by L, then the equation is written [2]:

$$\begin{aligned}&\alpha\left(\frac{T}{T^\circ} - \frac{2L}{RT}\,\frac{\varkappa-1}{\varkappa+1}\right)\left[1 - \left(\frac{T}{T^\circ}\right)^{1/2}\frac{p_s(T^\circ)}{p}\right]\\ &-\alpha\left[1 - \left(\frac{T^\circ}{T}\right)^{1/2}\frac{p_s(T^\circ)}{p}\right] + (1-\alpha)\beta\left(\frac{T^\circ}{T} - 1\right) = 0\end{aligned} \tag{1.2}$$

In Eqs. (1.1) and (1.2) the unknown coefficients α and β occur, which are determined experimentally. Their range of variation lies within the limits from 0 to 1. Quasi-one-dimensional calculations of the expansion of water vapor in nozzles [3] showed that by suitably choosing the coefficients α and β, the agreement between theoretical and experimental distributions of the static pressure was satisfactory. It was

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noted also that the coefficient of thermal adaptation has an insignificant effect on the static pressure distribution. For this reason, it has been assumed that $\beta = 1$ in these calculations.

In contrast from [1], where the state of flow was investigated along the lines of flow, in consequence of the characteristics of the experiment in this paper comparison of the static pressure has been made along lines parallel to the direction of the stream on the initial characteristic lines of the beam. The coefficient of condensation α was chosen in such a way that the theoretical and experimental pressure distribution along the horizontal lines and the wave pattern of the flow coincided as far as possible.

The flow field was calculated by the method of characteristic lines. The region of flow between the initial and closing characteristic lines of the beam, outgoing from the deflection point, was investigated. From above, this region is bounded by the first Mach line reflected from the upper wall of the nozzle.

2. The experiment was carried out at the Moscow Power Institute in the Faculty of Steam and Gas Turbines using an installation for studying two-phase flow [4]. A uniform supersonic flow was produced after expansion in a plane nozzle before flowing round the obtuse angle (Fig. 1). The experimental data show that expansion takes place almost without loss right up to the angular point, the flow ahead of it is uniform and there is no condensation. The parameters at the initial characteristic pattern outgoing from the deflection point were calculated by the formulas for a uniform isentropic expansion at a given temperature T_0 and pressure p_0 ahead of the nozzle. The isentropy index for water vapor was assumed to be 1.3 ($\varkappa = 1.3$).

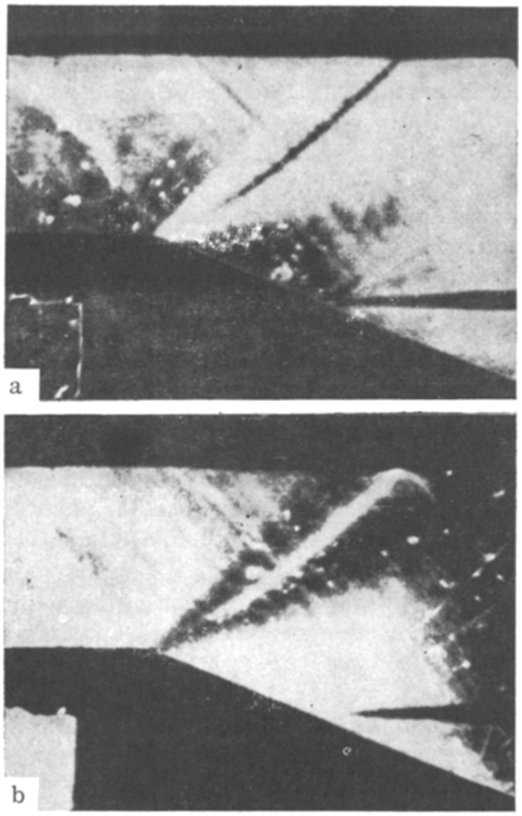

Fig. 1

The wave spectrum of the stream was recorded by means of the IAB-451 shadow equipment. The wave spectrum of one of the expansion cycles of the supersonic flow of supercooled water vapor is shown in Fig. 1. The shadow equipment records the zone of maximum density gradients and discriminates it in relation to the position of the blade by a dark (Fig. 1a) and light (Fig. 1b) line. Thus, the condensation discontinuities are defined experimentally as a narrow zone in which a sharp change of pressure or density is recorded. At the same time, the start of condensation is determined more precisely, theoretically [1] by the line of maximum supersaturation (or supercooling), which is located upwards of the flow from the zone of sharp change of density. If the condensation discontinuity zone is narrow, then both determinations almost coincide.

The parameters at the start of the rectilinear pattern of the beam $T_1 = 307°\mathrm{K}$, $p_1 = 0.328 \cdot 10^5\ \mathrm{N/m^2}$, and $M_1 = 1.35$ (where M is the Mach number) correspond to the parameters at the nozzle inlet, $T_0 = 390°\mathrm{K}$ and $p_0 = 0.935 \cdot 10^5\ \mathrm{N/m^2}$.

The static pressure distribution in the flow gas was investigated. As it was almost impossible to make the measurement of the parameters along the line of flow, the static pressure distribution was measured at various distances from the angular point in sections parallel to the advancing flow. The measurements were carried out with a pin-probe which could be moved in both longitudinal and transverse directions. The following sections were chosen for the measurements: $y/h = 0.14$, 0.45, and 0.61. The distance along the y axis between the deflection point and the upper wall of the nozzle was chosen as the characteristic value of h ($h = 0.37$ cm).

3. The supersonic flow field of water vapor in the region of the centered rarefaction wave formed by flow round an angle of 25° was calculated for a number of values of the coefficient of condensation α. A study of the wave spectrum of the flow and the static pressure distribution obtained experimentally made it possible to choose the value of the coefficient of condensation as $\alpha = 0.04$.

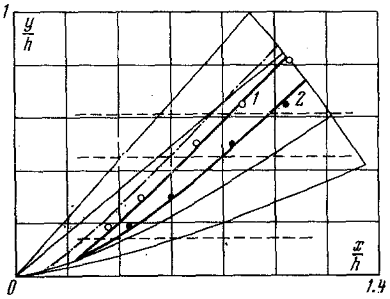

Fig. 2

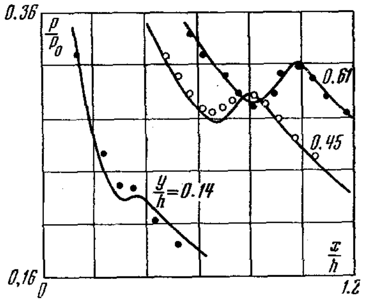

Fig. 3

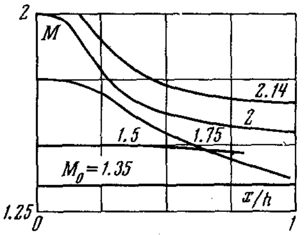

Fig. 4

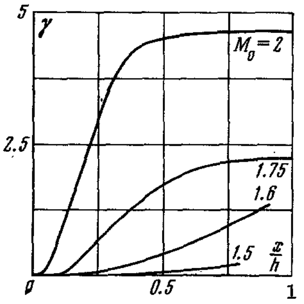

Fig. 5

Figure 2 shows a schematic diagram of the flow in a centered rarefaction wave. The thin lines drawn from the origin represent the characteristic curves of the first family. The same line, drawn from the point of intersection of the initial beam characteristic lines with the upper wall of the nozzle (the line y = h), indicates the first reflected characteristic curve of the second family, above which the calculation has not yet been carried out. This was because, in order to calculate the flow above the reflected Mach line, it is necessary to satisfy the supplementary boundary condition at the wall. The horizontal dashed lines mark the sections along which the static pressure measurements were carried out.

In Fig. 3 curves are drawn of the static pressure distribution along the dashed horizontal lines in Fig. 2. The results of the theoretical investigations are shown by the solid lines. The experimental data are plotted by small circles. It can be seen from the figure that a stronger pressure increase with distance from the angular point is observed in the condensation zone and the width of this zone increases. Such a nature of change of static pressure in the condensation zone is governed by the relation between the rate of expansion of the medium and the rate of supply of heat released by condensation. Expansion takes place with a decrease of static pressure, while the release of heat in the supersonic flow leads to an increase of the static pressure. Depending on this curve, the pressure distribution can have a different form [5]. In the vicinity of the angular point, the rate of expansion increases strongly but the rate of condensation varies less significantly which leads to a smoothing of the static pressure distribution curve in the zone of the condensation discontinuity.

For this reason, the condensation discontinuity can be recorded experimentally only at a certain distance from the angular point (Fig. 1). The width of the condensation discontinuity increases with distance from the point of deflection; its maximum value, however, does not exceed 7 mm in the case being considered. As shown in Fig. 3, the experimental data are found to be in satisfactory agreement with the theoretical calculations.

The curve on which supercooling reaches the maximum is plotted in Fig. 2 (chained). If this maximum is recorded along the lines of flow, then the curve will have a kink at its point of intersection with the closing line of the beam characteristic curve, after which it coincides with this characteristic curve up to the apex of the angle [1]. This curve can be considered as the leading front of the condensation zone. The heavy lines 1 and 2 in Fig. 2 correspond to lines of minimum and maximum pressure. Below the point of intersection of these lines, the curve of the change of static pressure along the horizontal y = const is changing monotonically. The experimental points taken from Fig. 1 are plotted here. The light circles correspond to the front line bounding the condensation discontinuity and the dark circles are the rear line. It can be seen that the results of the experiment are in satisfactory agreement with theory, as the shadow device records the region of maximum pressure gradients, i.e., the region between p_{min} and p_{max} in the condensation zone.

The characteristic lines emerging from the deflection point remain rectilinear almost until they no longer intersect the leading front of the condensation zone (chained curve), i.e., until the instant when condensation has a significant effect on the flow parameters. After entering the zone of the condensation discontinuity, the characteristic lines begin to twist and their angle of inclination to the x axis increases. After intersecting the line of maximum pressure (curve 2 in Fig. 2) the slope of the characteristic lines changes less intensely, according to the degree of penetration into the region where expansion takes place with a small deviation from equilibrium. At sufficiently large distances from the condensation discontinuity, the angle of inclination of the characteristic lines remains almost constant.

In the case being considered, despite the increase of the slope of the intersection, no characteristic lines were observed. It is interesting to note that the calculations carried out for larger values of the coefficient of condensation ($\alpha = 0.5$ and 1.0) recorded the intersection of the characteristic lines which, as is well known from gas-dynamics, should lead to the formation of a compression shock. The greater the value of the coefficient of condensation, the more intensely does the formation of nucleates and their growth take place and the greater the rate of supply of heat to the stream. This, in its turn, leads to stronger twisting of the characteristic lines. The results of calculation of the flow, obtained with $\alpha = 0.5$ and 1.0, do not agree with the experimental data. Agreement occurs when $\alpha = 0.04$ but in this case the characteristic lines do not intersect. Thus, in the case considered, despite the increase of pressure in a centered rarefaction wave the condensation discontinuity is not accompanied by shock compression. However, in principle the possibility of the appearance behind the condensation discontinuity of a shock compression should not be excluded. This problem requires additional study.

It is well known that when calculating supersonic flows of an ideal gas, the characteristic lines in a centered rarefaction wave are lines of constant parameters. This condition is not observed when expansion is accompanied by condensation. Figure 4 shows the behavior of the frozen Mach number along different characteristic lines of the beam. Values of the Mach numbers are shown for each curve on the characteristic line being considered at the angular point M_0. The change of specific moisture content γ along the characteristic lines is shown in Fig. 5. The figures above the curves denote the same as in Fig. 4. It can be seen from the graphs that the greatest change of parameters along the characteristic lines occurs in the zone of the condensation discontinuity. After crossing this zone the intensity of the change of parameters decreases and then they remain almost constant.

Agreement between theory and experiment was achieved by varying the coefficient of condensation α, for which the coefficient of thermal adaptation β was assumed to be equal to unity. Obviously, for a more reasonable choice of β, not only the wave pattern of the flow and the static pressure distribution must coincide, but also certain characteristics of the flow, for example dispersivity.

LITERATURE CITED

1. R. A. Tkalenko, Spontaneous Condensation in the Case of Supersonic Flow around a Protuberant Angle [in Russian], Izv. Akad. Nauk SSSR, Mekhan. Zhidk. i Gaza, No. 5, 73-77 (1970).
2. Kan' San-Vuk, "Investigation of the growth of condensed particles in rarefied and continuous media," Raketnaya Tekhnika i Kosmonavtika, 5, No. 7, 91-99 (1967).
3. V. P. Bakhanov and M. V. Buikov, "Kinetics of quasi-stationary homogeneous condensation of water vapor in a supersonic nozzle," in: Physics of Aerodispersed Systems [in Russian], No. 2, Kiev (1970).
4. M. E. Deich, V. F. Stepanchuk, G. A. Saltanov, and G. V. Tsiklauri, "Experimental investigation of condensation discontinuities," Teplofiz. Vys. Temp., 2, No. 5, 789-796 (1964).
5. M. E. Deich and G. A. Filippov, Gas Dynamics of Two-phase Media [in Russian], Énergiya, Moscow (1968).